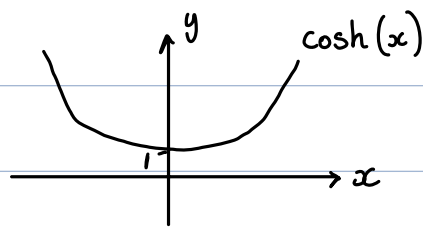
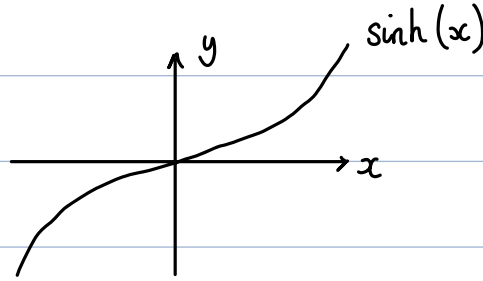


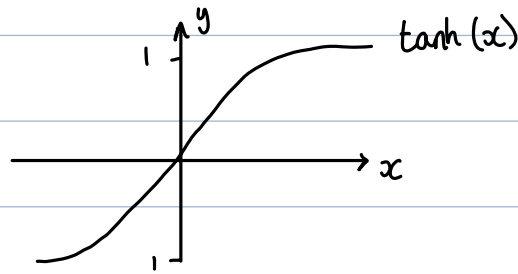
$$\cosh = \frac{1}{2} (e^x + e^{-x})$$



$$\sinh = \frac{1}{2} (e^x - e^{-x})$$



$$\tanh = \frac{\sinh}{\cosh}$$



$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh(A+B) = \sinh(A)\cosh(B) + \cosh(A)\sinh(B)$$

$$\cosh(A+B) = \cosh(A)\cosh(B) + \sinh(A)\sinh(B)$$

Graph of $\cosh(x)$ called a catenary:

- Shape of chain hanging under gravity
- Used in structural design

$$e^{+j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\therefore \cos\theta = \frac{1}{2} (e^{+j\theta} + e^{-j\theta})$$

$$\sin\theta = \frac{1}{2j} (e^{+j\theta} - e^{-j\theta})$$

$$\text{let } \theta = jx$$

$$\cos jx = \frac{1}{2} (e^x + e^{-x}) = \cosh x$$

$$\sin jx = \frac{1}{2j} (e^{-x} - e^x) = j \sinh x$$

$$\tan jx = j \tanh x$$

$$\cos^n \theta = \left[\frac{1}{2} (e^{j\theta} + e^{-j\theta}) \right]^n, \quad \sin^n \theta = \left[\frac{1}{2j} (e^{j\theta} - e^{-j\theta}) \right]^n$$

Complex Arguments for sin, cos, etc.

$$\sin(z) = \sin(x + jy)$$

$$= \sin(x) \cos(jy) + \cos(x) \sin(jy)$$

$$\text{so } \sin(z) = \sin(x) \cosh(y) + j \cos(x) \sinh(y)$$

$$\text{and } \cos(z) = \cos(x) \cosh(y) - j \sin(x) \sinh(y)$$

find values of z such that $\cos z = 2$

$$\cos(x+jy) = 2$$

$$\cos(x) \cosh(y) - j \sin(x) \sinh(y) = 2 + j0$$

$$\textcircled{1} \quad \cos(x) \cosh(y) = 2 \quad \textcircled{2} \quad \sin(x) \sinh(y) = 0$$

from $\textcircled{2}$ either $y = 0$ or $x = 0$